Understanding the Implications of Higher Inflation Targets *

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Abstract

A recent monetary policy proposal is that central banks should pursue a higher inflation target that would give them more "room-to-manoeuvre" in deep recessions and avoid costly periods at the effective lower bound (ELB) for nominal interest rates. This paper documents a non-trivial trade-off of such an approach: in staggered-price-setting models, a higher level of inflation increases its volatility and gives rise to more volatile policy rates if monetary policy is accommodative. Trend inflation emphasizes the importance of future marginal cost of current price setters and amplify the effects of economic shocks. Using a standard New Keynesian model, I revisit the frequency and average duration at the ELB. I show that a higher inflation volatility can have a first-order effect and ELB episodes can, in fact, become *more frequent*. The findings in this paper suggest that the ELB would bind around 10% more of the time if the inflation target is raised from 2% to 4%. I use a global solution method for the model in a fully stochastic setting to show that the probability of going to the ELB and the average expected duration of a liquidity trap increase under a 4% inflation target. The results are sensitive to (i) the interest-rate sensitivity of aggregate demand (via the Euler equation) and (ii) the degree of price rigidity in the model. Inflation volatility under positive trend inflation is higher the (i) lower the interest-rate sensitivity, and the (ii) higher the degree of price rigidities.

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1 Introduction

Figure 1 shows the evolution of the nominal effective Federal Funds Rate, inflation and an estimate of the real natural interest rate. A number of studies have documented a decline in the natural real rate (e.g. Del Negro et al. (2017), Hamilton et al. (2015), Gagnon et al. (2016), and Eggertsson et al. (2017)). A lower steady state real rate has large implications for monetary policy. Given that inflation has been fairly stable in the past 20 years, lower steady state real rates will be associated with lower steady state nominal interest rates. One potential consequence of lower equilibrium nominal rates is that episodes at the effective lower bound (ELB) on nominal interest rates would become more frequent, hampering the ability of monetary policy to stabilize the economy and deteriorating economic activity and inflation.

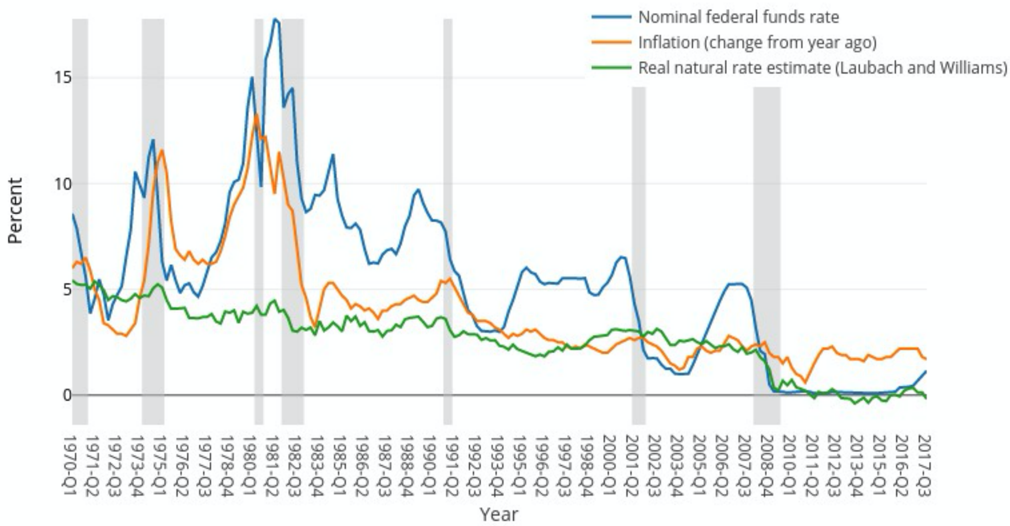

Figure 1: The Nominal FED Funds Rate, Inflation and the Real Natural Rate

Some researchers have advocated to raise the central bank's inflation target to increase the steady state nominal interest rate. For example, Blanchard et al. (2010) and Ball (2013) propose a target inflation rate of 4%. Kiley and Roberts (2017) study the frequency and potential costs of such episodes in a low interest-rate environment in a dynamic-stochastic-general equilibrium (DSGE) model and a large-scale econometric model, the FRB/US model. Their findings suggest that a higher steady state nominal interest rate can significantly reduce the frequency of ELB episodes and the mean duration at the ELB.

Others have argued against such a proposal. In his 2010 Jackson Hole speech, Ben Bernanke observed that, "Inflation expectations appear reasonably well-anchored, and both inflation expectations and actual inflation remain within a range consistent with price stability. In this context, raising the inflation objective would likely entail much greater costs than benefits. Inflation would be higher and probably more volatile under such a policy, undermining confidence and the ability of firms and households to make longer-term plans, while squandering the Fed's hard-won inflation credibility. Inflation expectations would also likely become significantly less stable, and risk premiums in asset markets-including inflation risk premiums-would rise."

Staggered-price-setting models typically predict a positive relationship between the level and the volatility of inflation. For example, Kiley (2007), Ascari and Sbordone (2014) and Blanco (2018) document that trend inflation amplifies the impact of macroeconomic shocks on inflation and output in a New Keynesian model. If monetary policy is accomodative, nominal interest rates can also become more volatile. This observation gives rise to a non-trivial trade-off of higher inflation targets: Positive trend inflation may lift up the equilibrium nominal rates which increases the central bank's ability to reduce its policy rate in recessions before it hits an ELB. All else equal, this could reduce the probability of ELB episodes. However, higher levels of inflation increase the volatility of inflation and the policy rate and would work against the first channel.

The following thought experiment illustrates the relationship between the steady state nominal interest rate and the probability of observing ELB events. Figure 2 shows probability densitiy functions of a normally distributed variable. Over the period from 1970 to 2007, the nominal FED funds rate averaged 6.6 percent. Suppose the realization of the FED funds rate can be represented by the probability density function with the dashed black line with mean 6.6. The standard deviation is calibrated such that the zero lower bound is expected to bind 5 percent of the time. The second probability density function (purple solid line) shows possible realizations of the policy rate under a higher inflation target. A higher inflation target is associated with a higher average nominal rate. However, we may also observe that inflation and the policy rate become more volatile as trend inflation increases. The latter can have a first-order effect so that the probability of reaching the ELB increases even though the mean of the nominal interest rate is higher.

This paper asks the following key question: Does higher inflation target necessarily help to avoid costly ELB episodes? To answer the question, I use a standard New Keynesian model which allows for an zero lower bound on the nominal interest rate. I find that higher trend inflation (i) increases the volatility of inflation and the policy rate and (ii) can lead to more frequent ELB episodes. Both observations are sensitive to the degree of interest-rate sensitivity of aggregate demand and the degree of price ridigities in the model economy. Inflation volatility increases the lower the interest-rate sensitivity and the higher the degree of price ridigities. This paper points to the possibility that higher trend inflation may not streghten but weaken the

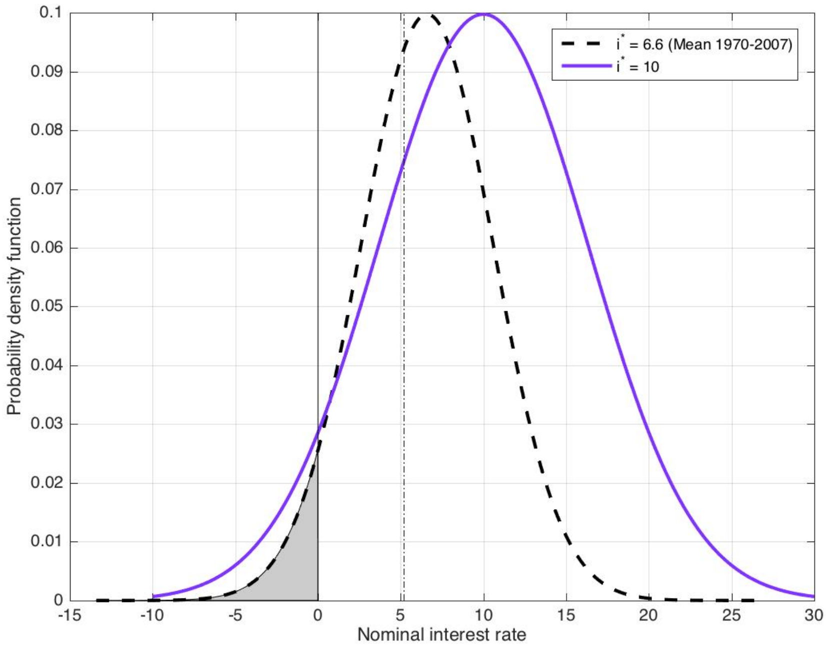

Figure 2: Probability density functions for alternative steady state nominal interest rates

capacity of central bank to stimulate the economy in a recession.

In a model with Calvo (1983) price-setting rigidities, positive trend inflation would imply higher inflation volatility for the following reason. In such a model, firms are forward-looking and anticipate that inflation will erode their mark-up over time (Ascari and Sbordone (2014)). As a consequence, firms tend to "overadjust" its reset price to compensate for the expected fall in real profits. In general equilibrium, fluctuations in prices are larger if the elasticity of private spending to changes in the real interest rate is low (via the New IS-curve). Using a global solution method to solve the model, I find that increasing the inflation target from 2 % to 4 % annually would increase the average frequency of ELB episodes by 10 percent and increase the mean duration of a liquidity trap by 1 quarter.

The paper is organized as follows. The next section discusses the empirical relationship between the level and the variance of inflation. Section 3 presents a standard New Keynesian model illustrating the effect of trend inflation on the price setting behavior of firms. Section 4 reports the frequency and duration of ELB events for inflation target rates ranging between 0% and 4%. Finally, Section 5 concludes.

2 Some Evidence: The Level and the Variance of Inflation

The data suggest that the level and variance of inflation are highly correlated. For example, Kiley (2000, 2007), Okun (1971) and Taylor (1981) have documented a positive relationship between the level and the variance of inflation. Based on an international cross-sectional comparison, these authors show that higher average inflation is associated with more volatile inflation.

Table 2 reports average inflation rates, as measured by the annual percent change in consumer prices (all non-food and non-energy items), and the the standard deviation of inflation for the G-7 countries over two periods, 1970-1985 and 1986-2007. The break in the time series in the late 1980s corresponds to the period after disinflation from the higher levels observed in the 1970s was completed (McConnell and Perez-Quiros (2000), OECD (2002), Kiley (2007)). The standard deviations for inflation are lower for all G-7 economies during the moderate inflation conditions from 1986 to 2007.

	1970-1985		1986-2007	
	Average	Std. Deviation	Average	Std. Deviation
Canada	7.1	2.7	2.6	1.6
France	9.3	2.6	2.1	1
Germany	4.4	1.4	2	1.3
Italy	13.3	5	3.8	1.8
Japan	6.9	4.6	0.7	1.1
United Kingdom	11.2	5	2.9	2.5
United States	6.7	2.7	3	1
G7	7.6	2.4	2.6	1.1

Table 1: Statistics for Consumer Price Inflation (No Food and Energy) in the G-7 (Source: OECD)

The relationship between low and more-stable inflation is also observable across countries. Figure 1 presents a scatter plot of country/time pairs for 20 countries and a G-7 measure for average inflation and its standard deviation[1]. The correlation coefficient between the average level and the standard deviation of inflation is 0.93 for 1970 to 2017. The strong positive correlation is not driven by the high inflation period in the 1970s and early 1980s. It remains high and nearly unchanged from 1986 to 2007 and to 2017, respectively.

Taylor (1981) suggests that accomodative monetary policies can lead to high inflation and greater variability in response to economic shocks[2]. It might be therefore possible that the rela-

[1] The countries are: Australia, Austria, Belgium, Canada, Denmark, Germany, France, Finland, Greece, Ireland, Italy, Japan, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, United Kingdom and United States.

[2] Another hypothesis for the relationship between the level and the variance of inflation was offered by Friedman (1977). He argues that high inflation may cause inefficient and more variable macroeconomic policies. However, his conjecture cannot explain the positive correlation between the average level of inflation and its variance at the moderate levels observed in the past 30 years.

tionship between the level of inflation and its variance mainly reflects the response of monetary policies in these countries.

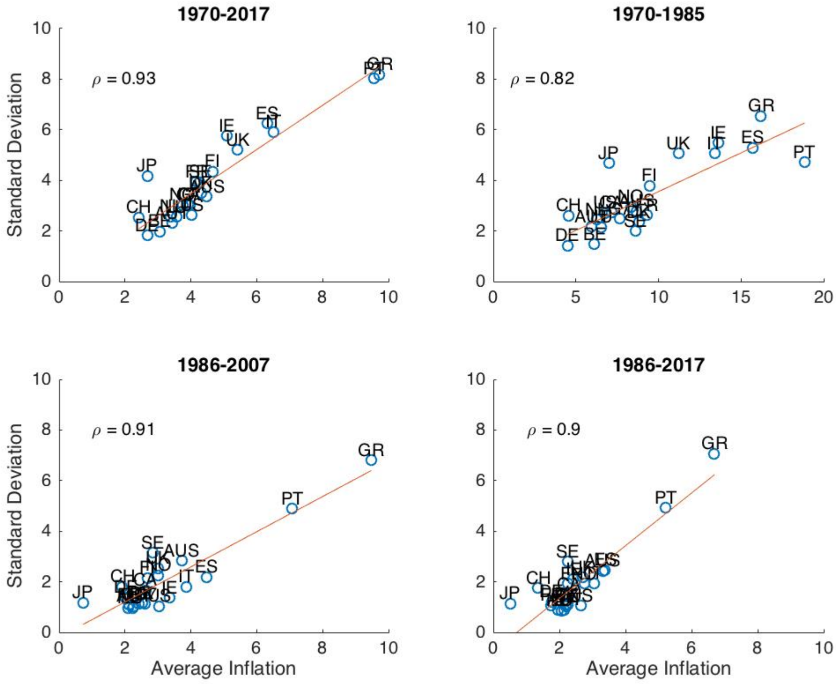

Figure 3: Average Inflation vs. Standard Deviation of Inflation for 20 Selected Countries

I re-examine the previous findings for countries of the eurozone. Since January 1999, the European Central Bank has been responsible for a single monetary policy in the euro area. Table 2 reports the average level and the standard deviation of consumer prices for all non-food and non-energy items for 11 European countries that launched the euro in 1999. Inflation appears more stable after 1999. The relationship between the level of inflation and its standard deviation is presented in Figure 4. The correlation is very high (0.94) before 1999. It remains positive after 1999 but is notably lower (0.5 between 1999-2007 and 0.59 between 1999-2017). While the positive link between inflation and its variance is less pronounced when we account for a single monetary policy, we can still observe that higher average inflation is associated with more volatile inflation.

The analysis in this paper is in line with Taylor's (1981) conjecture on the role of monetary policy. However, it will emphasize how a higher level of trend inflation can itself amplify the effects of economic shocks establishing a positive relationship between inflation and its variance as seen in the data – independent of monetary policy.

	1986-1999		1999-2007	
	Average	Std. Deviation	Average	Std. Deviation
Austria	3	1.1	1.5	0.5
Belgium	3.3	1.4	1.7	0.3
Finland	4.1	2.3	1.2	0.9
France	3.3	1.6	1.2	0.6
Germany	2.6	1.4	1.2	0.4
Ireland	3.5	1.8	3.2	1.6
Italy	6	2.1	2.2	0.3
Luxembourg	2.9	1.5	1.7	0.5
Netherlands	2.2	0.9	2.2	0.7
Portugal	12.2	6	3.3	1
Spain	6.5	2.3	2.7	0.3

Table 2: Statistics for Consumer Price Inflation (No Food and Energy) for Selected Euro Countries (Source: OECD)

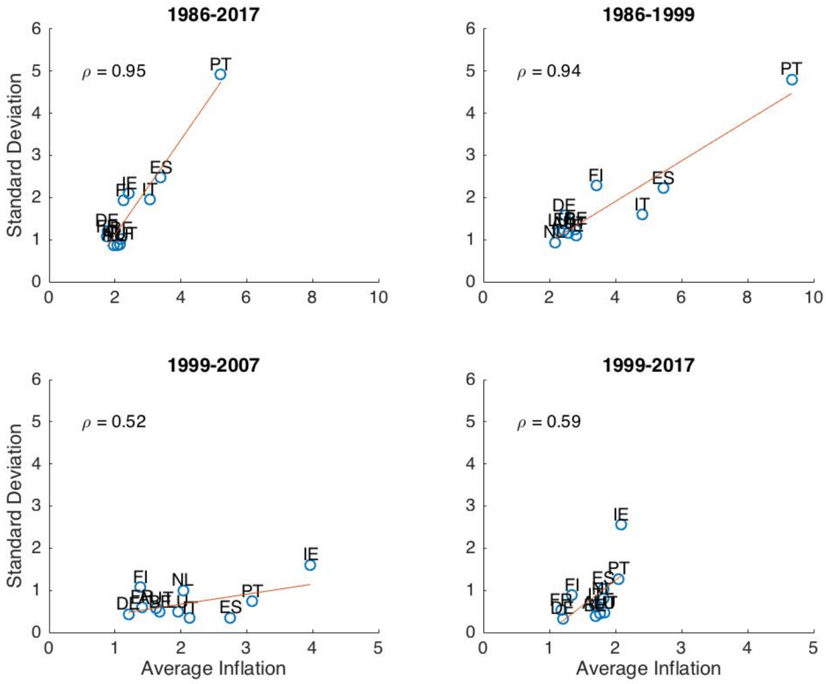

Figure 4: Average Inflation vs. Standard Deviation of Inflation for 11 Eurozone Countries

3 The Cost of Raising the Inflation Target

The analysis focuses on a stylized New Keynesian model with inflation target $\overline{\pi}$. Let π_t be (gross) inflation and y_t be output. R_t is the (gross) nominal interest rate. In the model economy, firms face Calvo (1983) price-setting rigidities. s_t be a measure of dispersion in relative prices. Price dispersion is a pivotal characteristic of price-staggered models and can be interpreted as the cost of inflation[3]. The log-linearized New Keynesian model with inflation target $\overline{\pi}$ can be expressed in terms of the following equations:

$$\text{New IS-curve:} \quad \widehat{y}_t = E_t\widehat{y}_{t+1} - \sigma\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) - \widehat{\xi}_t \tag{1}$$

$$\text{Phillips Curve:} \quad \widehat{\pi}_t = \beta\omega(\overline{\pi})E_t\widehat{\pi}_{t+1} + \kappa(\overline{\pi})[\widehat{y}_t + \widehat{s}_t] + \eta(\overline{\pi})E_t\widehat{\psi}_{t+1} \tag{2}$$

$$\text{with} \quad \widehat{\psi}_t = b(\overline{\pi})\left((\varphi+1)\widehat{y}_t + \varphi\widehat{s}_t\right) + (1-b(\overline{\pi}))E_t\left(\widehat{\psi}_{t+1} + \widehat{\pi}_{t+1}\right) \tag{3}$$

$$\text{Price dispersion:} \quad \widehat{s}_t = c(\overline{\pi})\widehat{s}_{t-1} + d(\overline{\pi})\widehat{\pi}_t \tag{4}$$

$$\text{Taylor rule:} \quad \widehat{R}_t = \max\left\{-\log(\overline{\pi}/\beta), \phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t\right\} \tag{5}$$

where $\widehat{}$ denote log-deviations from steady state. The derivation of the New Keynesian Phillips curve and the equation for price dispersion are shown in the appendix[4]. $\widehat{\xi}_t$ is a shock to the New IS-curve. For example, $\widehat{\xi}_t$ can be nore generally interpreted as a discount factor shock (e.g. Christiano et al. (2011)). It follows an AR(1) process:

$$\widehat{\xi}_t = \rho\widehat{\xi}_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2) \tag{6}$$

Equation (1) relates current output positively to expected future output and inversely to the real interest rate $\widehat{R}_t - E_t\widehat{\pi}_{t+1}$. σ can be interpreted is the interest rate elasticity of aggregate demand, capturing intertemporal substitution in private spending. σ will play a key role in the analysis.

Equation (2) is the generalized New Keynesian Phillips curve for inflation target $\overline{\pi}$. $\omega(\overline{\pi})$, $\kappa(\overline{\pi})$ and $\eta(\overline{\pi})$ are parameters that are set as functions of the inflation target $\overline{\pi}$. Note that $\widehat{s}_t \approx 0$ if $\overline{\pi} = 1$. Small perturbations around the steady state have zero first-order effect on $\widehat{s}_t$ because the model is log-linearized around a point at which there is no price dispersion (Ascari and Sbordone (2014)). ψ_t is an auxiliary variable and can be represented recursively (in log-deviations) as in Equation (3). Equation (4) is the expression for price dispersion.

[3]Typically, s_t can be represented as resource cost: the higher the dispersion in relative prices, the higher is the labor input needed to produce a given amount of output (Yun (1996)).

[4]Note that if the inflation target is zero, i.e. $\overline{\pi} = 0$, the Phillips curve reduces to the familiar expression: $\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \kappa\widehat{y}_t$.

Finally, Equation (5) is a simple Taylor rule that describes how the nominal interest rate adjusts to deviations of inflation from target and of output from its steady state but is constrained by a zero lower bound. [5].

3.1 Calibration

Positive trend inflation alters the price dynamics in response to shocks. In order to illustrate and understand how higher inflation targets affect the volatility in inflation and in the policy rate, I consider several numerical experiments. To perform the experiments, I use the following parameter values: I set the quarterly discount factor β to 0.988. For an inflation target rate of 2% annually, this would imply a steady state nominal interest rate of 6.8% (annually) which would roughly be in line with the average nominal FED Funds rate between 1970 and 2007. We have the following expressions for the parameters in the model:

$$\omega(\overline{\pi}) = 1 + \epsilon(\overline{\pi} - 1)(1 - \theta\overline{\pi}^{\epsilon-1}) \tag{7}$$

$$\kappa(\overline{\pi}) = \frac{(1 - \theta\beta\overline{\pi}^{\epsilon})(1 - \theta\overline{\pi}^{\epsilon-1})}{\theta\overline{\pi}^{\epsilon-1}} g(\varphi) \tag{8}$$

$$\eta(\overline{\pi}) = -\beta(1 - \overline{\pi})(1 - \theta\overline{\pi}^{\epsilon-1}) \tag{9}$$

$$b(\overline{\pi}) = 1 - \theta\beta\overline{\pi}^{\epsilon} \tag{10}$$

$$c(\overline{\pi}) = \theta\overline{\pi}^{\epsilon} \tag{11}$$

$$d(\overline{\pi}) = \frac{\epsilon\, \theta\overline{\pi}^{\epsilon-1}}{1 - \theta\overline{\pi}^{\epsilon-1}}(\overline{\pi} - 1) \tag{12}$$

The Calvo parameter θ, which is the probability of not-resetting a price in a given period, is set to 0.75. The inverse Frisch-elasticity, φ, is chosen to be 1. Assume for simplicity that $g(\varphi) = 1$. The elasticity of substitution among intermediate inputs, ϵ, is set to 6, assuming a steady state mark-up for firms of $\frac{\epsilon}{\epsilon-1} = 1.2$. The inflation response coefficient, ϕ_π, in the Taylor rule is 1.5 and the output response, ϕ_y, is 0.125. Finally, the AR(1)-coefficient of the shock to the IS-curve, ρ, is set to 0.85. These parameter values are fairly standard in the literature.

3.1.1 The steady state nominal interest rate

The steady state nominal interest rate increases with higher inflation target rates. The equilibrium (gross) nominal rate on quarterly basis is: $\overline{R} = \overline{\pi}/\beta$. Let $\beta = 0.988$, Table 3 summarizes the steady state values for the (annual) steady state nominal interest rates.

[5] In the log-linearized model, the nominal interest rate is zero if $\widehat{R}_t = -\log(\overline{\pi}/\beta)$ where R_t is the gross nominal rate.

Inflation Target (Annual)	Nom. Interest Rate (Annual)
0%	4.8%
2%	6.8%
3%	7.9%
4%	8.9%

Table 3: Steady state nominal rates

Higher inflation targets lift up the steady state nominal rate and creates a larger leeway for the central bank to cut the policy rate before the zero lower bound is hit.

3.1.2 Price setting dynamics

A higher inflation target changes the parameters in the Phillips curve $\omega(\overline{\pi})$ and $\kappa(\overline{\pi})$ in opposite directions. In fact, a higher inflation target makes the slope of the New Keynesian Phillips curve flatter ($\kappa(\overline{\pi})$ goes down) while the weight on expected inflation increases ($\omega(\overline{\pi})$ goes up). Table 4 shows the values for $\omega(\overline{\pi})$ and $\kappa(\overline{\pi})$ for different levels of the inflation target.

Inflation Target (Annual)	$\omega(\overline{\pi})$	$\kappa(\overline{\pi})$
0%	1	0.086
2%	1.007	0.071
3%	1.01	0.064
4%	1.0127	0.057

Table 4: Parameters in New Keynesian Phillips Curve

The rationale for the observation in Table 4 is the following. Consider the price setting problem of the optimizing firm[6]. Let p_t^* be the optimal re-set price:

$$p_t^* = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} y_{t+j} \Pi_{t,t+j}^{\epsilon} mc_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} y_{t+j} \Pi_{t,t+j}^{\epsilon-1}} \tag{13}$$

where $\Lambda_{t,t+j}$ is a stochastic discount factor and mc_t is marginal cost. $\Pi_{t,t+j}$ denotes the cumulates gross inflation rate over j periods:

$$\Pi_{t,t+j} = \begin{cases} 1 & \text{for } j = 0 \\ \pi_{t+1} \times ... \times \pi_{t+j} & \text{for } j = 1, 2, ... \end{cases} \tag{14}$$

Forward-looking firms anticipate that they may be stuck with the price set in period t and that inflation will erode their mark-up over time (Ascari and Sbordone (2014)). They effectively use future expected inflation rates to discount future marginal cost. The higher the expected future

[6] See appendix for details.

inflation rates, the larger the weight on expected future marginal costs. Firms become more forward-looking and place more weight on expected future conditions rather than on current conditions. Intuitively, firms tend to "overadjust" its reset price to compensate for the expected fall in real profits. This is a consequence of their inability to set the optimal price in the upcoming periods and the erosion of real mark-up due to positive trend inflation.

3.1.3 The interest-rate elasticity of aggregate demand

I set the interest-rate elasticity in the New IS-curve, σ, to 0.25 and, hence, assume a relatively flat slope of the the dynamic IS equation. Eggertsson and Woodford (2003) specify $\sigma = 0.5$[7]. The degree of interest sensitivity will play an important role for inflation volatility under positive trend inflation in general equilibrium. Assuming that prices are sufficiently rigid, higher steady state nominal rates may enable the central bank to stabilize the economy by reducing the real interest rate even further in response to contractionary shocks. If aggregate demand is sensitive to changes in the real rate (high σ), any contraction in output and fall in prices will be dampened in equilibrium. The demand channel reduces the volatility of inflation in general equilibrium again. I will discuss the relative standard deviation of inflation for a target of 4 % vs. 2 % in Section 3.3. It can be noted here, that a lower σ is associated with higher relative standard deviations.

3.2 Solution Method

I solve the log-linearized model using a global solution method. This solution method discretizes the state space and uses fixed point iteration to solve for the updated decision rules until the tolerance criterion is met. The appendix provides further details. The only shock that is considered in the numerical experiments is the shock to the IS-curve (i.e. discount factor shock). For the standard deviation of the shock, σ_ξ, I set 0.55/100 which would make the frequency of hitting the ZLB around 10 percent at inflation target 0%.

3.3 The Numerical Experiments

To illustrate the implications of higher inflation targets, I consider the following experiment. In the first scenario, I assume that the model economy is subject to a positive discount factor shock. The discount factor shock is contractionary but not large enough to put the economy into

[7] Eggertsson and Woodford (2003) and explain their choice as follows: "We prefer to bias our assumptions in the direction of only a modest effect of interest rates on the timing of expenditure, so as not to exaggerate the size of the output contraction that is predicted to result from an inability to lower interest rates when the zero bound binds." (Footnote 36)

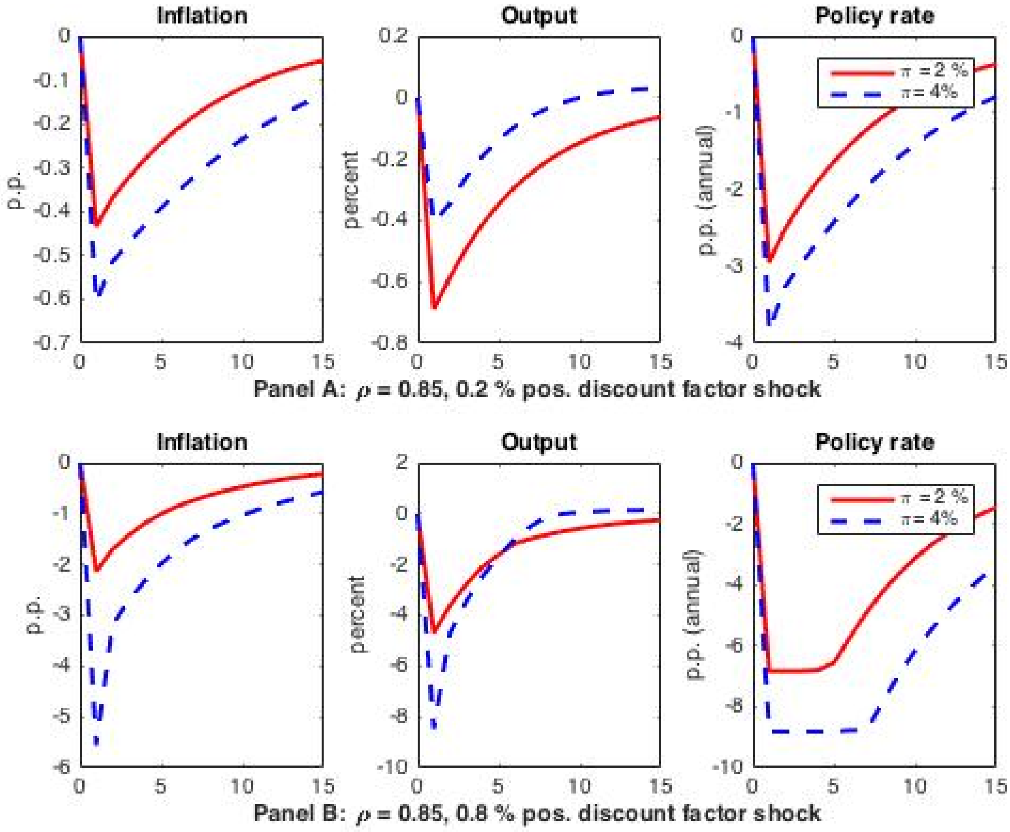

Figure 5: Impulse responses after discount factor shock

a liquidity trap. I consider two inflation targets: 2 % and 4% annually [8]. Panel A of Figure 5 shows the impulse responses for selected variables to the discount factor shock.

The higher target inflation of 4 % amplifies the impact of the shock on inflation because price-setting firms are more forward-looking. It also amplifies the response of the policy rate. Furthermore, we observe that a stronger cut in the real rate; hence, the contraction in output is dampened. In other words, a higher inflation target exarcebate the fall in prices, nominal and real rates as long as the economy does not hit the zero lower bound. In this case, higher trend inflation reduces the contraction in output.

Panel B of Figure 5 depicts impulse responses to a larger contractionary discount factor shock that generates a liquidity trap of 4 quarters under a 2 % inflation target. Again, higher trend inflation amplifies the fall in prices and the policy rate. However, in this case, the response of inflation and the policy rate are strong enough to hit the zero lower bound. With the nominal interest rate stuck at the zero bound, the stronger fall in prices drives up the real interest rate and further contracts private spending. The drop in output leads to a further cut in prices and

[8]This would correspond to $\overline{\pi} = 1.005$ and $\overline{\pi} = 1.01$ on a quarterly basis.

		θ		
		0.5	0.65	0.75
σ	1	0.16	0.85	1.27
	0.5	0.35	0.93	1.29
	0.25	0.84	1.35	1.83

Table 5: Relative standard deviations of inflation for inflation target 4 % vs. 2 %

expected inflation and a further rise in the real rate. The net effect is a large decrease in output and inflation. Effectively, a higher inflation target amplifies the deflationary spiral associated with the zero-lower bound.

Table 5 shows the relative standard deviations of inflation for an inflation target of 4 % vs. 2 % for different parameter specifications for the Calvo parameter θ and the interest-rate elasticity σ [9]. For a given σ, the relative standard deviation increases with the degree of price rigidity. When the probability of not-resetting becomes larger, firms effectively become more forward-looking everything equal. In combination with a higher inflation target, firms pay even more attention to future expected marginal cost rather than to current economic conditions.

For a given θ, the relative standard deviation decreases as the interest-rate elasticity goes up. The higher σ, the stronger demand reacts to changes in the real rate which, in turn, reduces the fall in prices and, hence, the volatility of inflation in general equilibrium.

The numerical experiments in this section have shown that a higher inflation target bears a non-trivial trade-off for the probability of hitting the zero lower bound for the policy rate. On the one hand, a higher inflation target would increase the steady state nominal interest rate which would give the central bank more scope to cut the policy rate before hitting an effective lower bound. However, on the other hand, a higher inflation target increases the volatility of inflation and, hence, of the policy rate. The numerical example in Figure 5 has shown that the higher volatility of inflation can have a first-order effect: a sufficiently large discount factor shock drives the economy into a longer liquidity trap for an inflation target of 4 %.

4 The Frequency and Duration of ELB episodes

I solved the model for a range of possible discount factor shocks under the calibration described in the previous section. Figure 6 plots the expected probability of being at the effective lower bound in the current period for different levels of the inflation target. The dashed lines represents

[9] I generate 6000 simulated samples of 150 periods from random draws of the normally distributed discount factor shock with an unconditional standard deviation $\sigma_\xi = 0.55/100$ and compute the empirical standard deviation of inflation. Note that the simulations for both inflation targets of 2 % and 4% are done assuming $\sigma_\xi = 0.55/100$.

the threshold shock that is actually needed to hit the zero lower bound. The ELB binds if the economy is hit by a shock that is larger than the threshold shock. Figure 6 shows that it would require a larger shock to take the economy to the zero lower bound if the inflation target is raised from 0 % to 2 % or to 3 %. The gain becomes smaller when moving from 2 % to 3 %. For an inflation target of 4 %, the implied volatility of inflation becomes a key driver so that a smaller shock than under 2 % is sufficient to observe a ELB episode. For an economy with an inflation target rate of 4 %, it becomes more likely to hit a lower bound on nominal rates for any given shock. Note that the overall frquency of ELB episodes increased as well.

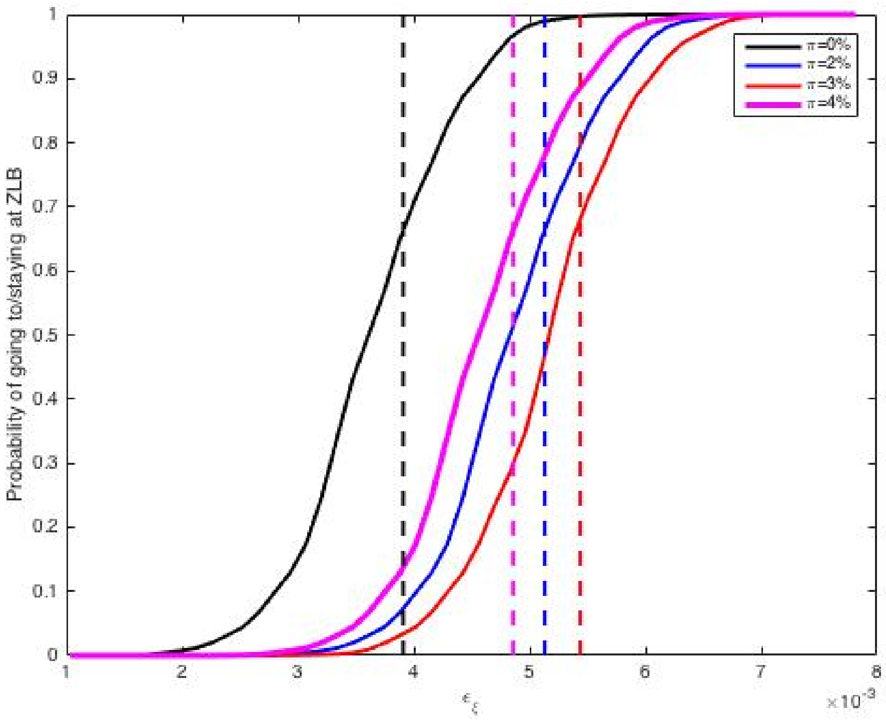

Figure 6: Probability of Going to/ staying at the ELB

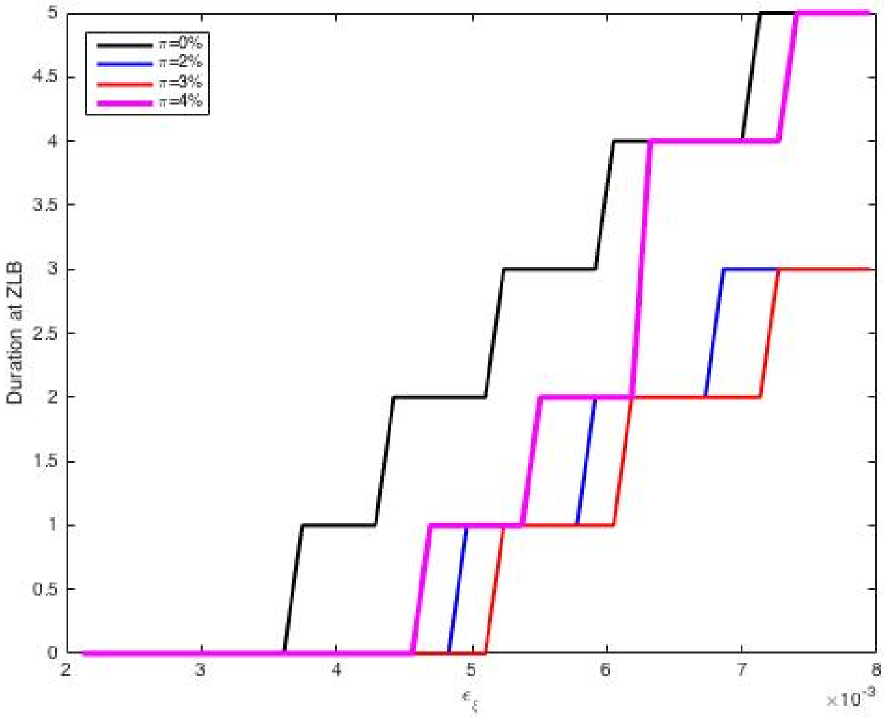

Figure 7: Average Duration at ELB

Figure 7 illustrates the average duration at the ELB for a range of possible shocks. Economic performance is worse under 4 % trend inflation. In cases where the lower bound binds, the expected duration of a liquidity trap is longer under an inflation target of 4 % compared to 2 %.

Under the chosen calibration, the New Keynesian model would predict that a higher inflation target does not necessarily reduces the frequency and duration of ELB events. Table 6 summarizes the results of the simulations. In fact, increasing the inflation target from 2 % to 4 % can increase the probability of a liquidity trap by 10 % and increase the expected mean duration at the ELB by around 1 quarter.

Inflation Target	Expected probability of ELB	Mean expected duration at ELB
0 %	11 %	3.15 quarters
2 %	8.3 %	2.6 quarters
3 %	7.1 %	2.62 quarters
4 %	9.1 %	3.53 quarters

Table 6: Economic Performance

5 Conclusion

[To be written.]

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A Appendix

A.1 Derivation of the New Keynesian Phillips Curve Under Positive Trend Inflation

Suppose that in each period a firm can reoptimize its nominal price with fixed probability $1-\theta$, while with probability θ it charges the price of the previous period. The firm i which reoptimizes its price in period t chooses $P^*_{i,t}$ to maximize expected profits:

$$E_t \sum_{j=0}^{\infty} (\theta\beta)^j \frac{\Lambda_{t+j}}{\Lambda_t} \left(P^*_{i,t} y_{i,t+j} - P_{t+j} TC_{t+j}(y_{i,t+j})\right) \tag{15}$$

subject to the demand function

$$y_{i,t+j} = \frac{P^*_{i,t}}{P_{t+j}}^{-\epsilon} y_{t+j} \tag{16}$$

where $\frac{\Lambda_{t+j}}{\Lambda_t}$ is the stochastic discount factor, $TC_{t+j}(y_{i,t+j}) = mc_{t+j} y_{i,t+j}$ is total cost and $mc_{t+j} = w_{t+j}$ is marginal cost.

Let $p^*_{i,t} = \frac{P^*_{i,t}}{P_t}$ be the relative price of the optimizing firm in period t. The first-order condition of the firm's problem can be written as:

$$p^*_{i,t} = \frac{\epsilon}{\epsilon-1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} y_{t+j} \Pi^{\epsilon}_{t,t+j} mc_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \Lambda_{t,t+j} y_{t+j} \Pi^{\epsilon-1}_{t,t+j}} = \frac{\epsilon}{\epsilon-1} \frac{\psi_t}{\phi_t} \tag{17}$$

where $\Pi_{t,t+j}$ denotes the cumulates gross inflation rate over j periods:

$$\Pi_{t,t+j} = \begin{cases} 1 & \text{for } j = 0 \\ \pi_{t+1} \times ... \times \pi_{t+j} & \text{for } j = 1, 2, ... \end{cases} \tag{18}$$

with $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate. We can rewrite ψ_t and ϕ_t recursively:

$$\psi_t = \Lambda_t mc_t y_t + \theta\beta E_t[\pi^{\epsilon}_{t+1} \psi_{t+1}] \tag{19}$$

$$\phi_t = \Lambda_t y_t + \theta\beta E_t[\pi^{\epsilon-1}_{t+1} \phi_{t+1}] \tag{20}$$

Let P_t be the price index associated with the final good y_t. It is a CES aggregate of the prices of the intermediate goods, $P_{i,t}$. It evolves as follows:

$$P_t = \left[\int_0^1 P^{1-\epsilon}_{i,t} di\right]^{\frac{1}{1-\epsilon}} = \left[\theta P^{1-\epsilon}_{t-1} + (1-\theta) P^{*1-\epsilon}_{i,t}\right]^{\frac{1}{1-\epsilon}} \tag{21}$$

Rearraging the equation above gives:

$$p^*_{i,t} = \left[\frac{1-\theta\pi_t^{\epsilon-1}}{1-\theta}\right]^{\frac{1}{1-\epsilon}} \tag{22}$$

Price dispersion between intermediate goods prices $P_{i,t}$ affects the relationship between employment, n_t and output y_t. Assume each firm produces with a linear production technology: $y_{i,t} = n_{i.t}$. Using the demand function in Eq. (16), aggregate labor demand is:

$$n_t = \int_0^1 n_{i,t} di \tag{23}$$

$$= \int_0^1 y_{i,t} di = y_t \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} di \tag{24}$$

Let s_t be a measure of price dispersion:

$$s_t = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} di, \tag{25}$$

aggregate output can be expressed as:

$$y_t = \frac{n_t}{s_t}. \tag{26}$$

Using Eq. (21), we can rewrite s_t:

$$s_t =(1-\theta)\left(\frac{P^*_{i,t}}{P_t}\right)^{-\epsilon} + \theta\pi_t^{\epsilon} \tag{27}$$

$$\times \left\{(1-\theta)\left(\frac{P^*_{t-1}}{P_{t-1}}\right)^{-\epsilon} + \theta^2(1-\theta)\left(\frac{P^*_{t-2}}{P_{t-1}}\right)^{-\epsilon} + ...\right\} \tag{28}$$

$$=(1-\theta)(p^*_{i,t})^{-\epsilon} + \theta\pi_t^{\epsilon} s_{t-1}. \tag{29}$$

We can now log-linearize Eqs. (17), (19), (20) and (22) around a deterministic steady state with trend inflation rate $\overline{\pi}$. Let $\widehat{}$ denote log-deviations from steady state. We do not model the household side explicitly. Assume for simplicity that $\widehat{\Lambda}_t = \widehat{y}_t$. We obtain the following expression for the dynamics of inflation:

$$\widehat{\pi}_t = \beta\omega(\overline{\pi})E_t\widehat{\pi}_{t+1} + \kappa(\overline{\pi})\widehat{mc}_t + \eta(\overline{\pi})E_t\widehat{\psi}_{t+1} \tag{30}$$

$$\text{with} \quad \widehat{\psi}_t = b(\overline{\pi})\left((\varphi+1)\widehat{y}_t + \varphi\widehat{s}_t\right) + (1-b(\overline{\pi}))E_t\left(\widehat{\psi}_{t+1} + \widehat{\pi}_{t+1}\right) \tag{31}$$

with

$$
\begin{aligned}
\omega(\overline{\pi}) &= 1 + \epsilon(\overline{\pi} - 1)(1 - \theta\overline{\pi}^{\epsilon-1}) \\
\kappa(\overline{\pi}) &= \frac{(1 - \theta\beta\overline{\pi}^{\epsilon})(1 - \theta\overline{\pi}^{\epsilon-1})}{\theta\overline{\pi}^{\epsilon-1}} \\
\eta(\overline{\pi}) &= -\beta(1 - \overline{\pi})(1 - \theta\overline{\pi}^{\epsilon-1}) \\
b(\overline{\pi}) &= 1 - \theta\beta\overline{\pi}^{\epsilon}
\end{aligned}
$$

Furthermore, assume the following labor supply equation: $\widehat{w}_t = g(\varphi)\widehat{n}_t$ and assume for simplicity that $g(\varphi) = 1$. Using Eq. (26), we can rewrite $\widehat{mc}_t$:

$$\widehat{mc}_t = \widehat{y}_t + \widehat{s}_t. \tag{32}$$

We can use the expression above to obtain the generalized New Keynesian Phillips curve:

$$\widehat{\pi}_t = \beta\omega(\overline{\pi})E_t\widehat{\pi}_{t+1} + \kappa(\overline{\pi})[\widehat{y}_t + \widehat{s}_t] + \eta(\overline{\pi})E_t\widehat{\psi}_{t+1} \tag{33}$$

Finally, we can use Eqs. (29) and (22) to show that price dispersion evolves as:

$$\widehat{s}_t = c(\overline{\pi})\widehat{s}_{t-1} + d(\overline{\pi})\widehat{\pi}_t \tag{34}$$

with $c(\overline{\pi}) = \theta\overline{\pi}^{\epsilon}$ and $d(\overline{\pi}) = \frac{\epsilon\theta\overline{\pi}^{\epsilon-1}}{1-\theta\overline{\pi}^{\epsilon-1}}(\overline{\pi} - 1)$. Note that in a zero inflation steady state, i.e. $\overline{\pi} = 1$, we have:

$$\widehat{s}_t = \theta\widehat{s}_{t-1}. \tag{35}$$

Hence, small perturbations have a zero first-order impact on $\widehat{s}_t$ (Ascari and Ropele (2009)). This implies that for $\overline{\pi} = 1$ that Eq. (33) becomes the familiar New Keynesian Phillips curve:

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \kappa\widehat{y}_t. \tag{36}$$

with $\kappa = \frac{(1-\theta\beta)(1-\theta)}{\theta}$.

A.2 Numerical Algorithm

The following section documents the numerical algorithm that used to solve the New Keynesian model. In particular, I use a collocation method to approximate the vector of decision rules, $\mathbf{Z}$ as a function of the state vector $\mathbf{s}$. I approximate $\mathbf{Z}$ by a linear combination of n basis functions

ϕ_i, $i = 1, ..., n$:

$$\mathbf{Z}(\boldsymbol{s}) \approx \widehat{\mathbf{\Phi}}(\boldsymbol{s})\mathbf{C} \tag{37}$$

where $\mathbf{C} = [\mathbf{c}^{\pi}\ \mathbf{c}^{\mathbf{y}}\ \mathbf{c}^{\psi}\ \mathbf{c}^{\mathbf{s}}]'$ and $c^j = [c_1^j\ ...\ c_n^j]$, $j = \{\pi, y, \psi, s\}$. Moreover, $\widehat{\mathbf{\Phi}}(\boldsymbol{s})$ is a block matrix:

$$\widehat{\mathbf{\Phi}}(\boldsymbol{s}) = \begin{bmatrix} \mathbf{\Phi}(\boldsymbol{s}) & 0 & \dots & 0 \\ 0 & \mathbf{\Phi}(\boldsymbol{s}) & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \mathbf{\Phi}(\boldsymbol{s}) \end{bmatrix} \tag{38}$$

where $\mathbf{\Phi}(\mathbf{s}) = [\phi_1(\boldsymbol{s}) \dots \phi_n(\boldsymbol{s}))]$. The coefficient vector C is set such that Eq. (39) holds exactly at n selected collocation nodes. Let the column vectors $\overline{\boldsymbol{\xi}}$ and $\overline{\boldsymbol{s}}$ contain the grid points of the discount factor shock and price dispersion, respectively. The vectors have length n_j, $j \in \{\xi, s\}$. The total number of grid points is $n = n_\xi \times n_s$. We can write compactly for matrix $\overline{\boldsymbol{S}}$ containing the grid points:

$$\mathbf{Z}(\overline{\boldsymbol{S}}) = \widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}})\mathbf{C} \tag{39}$$

where

$$\overline{\mathbf{S}} = \begin{bmatrix} [\mathbf{1}_{(n_s)} \otimes \overline{\boldsymbol{\xi}}]' \\ [\overline{\boldsymbol{s}} \otimes \mathbf{1}_{(n_\xi)}]' \end{bmatrix}' \tag{40}$$

$\overline{\mathbf{S}}$ is a matrix of dimension $n \times 4$ and $\not\Vdash_h$ is a column vector of ones of length h. $\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}})$ is again a block matrix

$$\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}) = \begin{bmatrix} \mathbf{\Phi}(\overline{\boldsymbol{S}}) & 0 & \dots & 0 \\ 0 & \mathbf{\Phi}(\overline{\boldsymbol{S}}) & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \mathbf{\Phi}(\overline{\boldsymbol{S}}) \end{bmatrix} \tag{41}$$

where

$$\mathbf{\Phi}(\overline{\boldsymbol{S}}) = \begin{bmatrix} \phi_1(\overline{\boldsymbol{S}}_{(1,:)}) & \dots & \phi_n(\overline{\boldsymbol{S}}_{(1,:)}) \\ \vdots & \dots & \vdots \\ \phi_1(\overline{\boldsymbol{S}}_{(n,:)}) & \dots & \phi_n(\overline{\boldsymbol{S}}_{(n,:)}) \end{bmatrix} \tag{42}$$

$\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}})$ has dimension $4n \times 4n$ and $\mathbf{\Phi}(\overline{\boldsymbol{S}})$ is $n \times n$. $\overline{\boldsymbol{S}}_{(k,:)}$ refers to the elements in row k of matrix $\overline{\boldsymbol{S}}$. Here, I use linear spline basis functions where the breakpoints coincide with the collocation nodes. This means that $\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}})$ becomes an identity matrix. Then, it holds:

$$\mathbf{Z}(\overline{\boldsymbol{S}}) = \mathbf{C} \tag{43}$$

I assume that the discount factor shock follows an AR(1) process: $\widehat{\xi}_t = \rho\widehat{\xi}_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. I use a Gaussian quadrature scheme to discretize normally distributed random variables. Assume that $\boldsymbol{\varepsilon} = [\varepsilon_1 \dots \varepsilon_m]'$ is a column vector with m quadrature nodes and $\boldsymbol{\omega} = [\omega_1 \dots \omega_m]'$ is a column vector of length m containing the quadrature weights. For the expected functions, the basis functions ϕ_i are evaluated for the matrix $\overline{\boldsymbol{S}}^E$:

$$\overline{\boldsymbol{S}}^E = \begin{bmatrix} [\mathbf{1}_{(n_s)} \otimes \overline{\boldsymbol{\xi}}^E]' \\ [\boldsymbol{c}^s \otimes \mathbf{1}_m]' \end{bmatrix}' \tag{44}$$

where

$$\overline{\boldsymbol{\xi}}^E = \begin{bmatrix} (\rho\overline{\xi}_1 \otimes \mathbf{1}_m) + \boldsymbol{\varepsilon} \\ \vdots \\ (\rho\overline{\xi}_{n_\xi} \otimes \mathbf{1}_m) + \boldsymbol{\varepsilon} \end{bmatrix}$$

Now, we can write the expected functions compactly as follows:

$$EZ(\overline{\boldsymbol{S}}) = \widehat{\mathbf{\Omega}}\, \widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^E)\, \boldsymbol{C} \tag{45}$$

where

$$\widehat{\mathbf{\Omega}} = \begin{bmatrix} \mathbf{\Omega} & 0 & \dots & 0 \\ 0 & \mathbf{\Omega} & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \mathbf{\Omega} \end{bmatrix} \qquad \mathbf{\Omega} = \begin{bmatrix} \boldsymbol{\omega}' & 0 & \dots & 0 \\ 0 & \boldsymbol{\omega}' & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \boldsymbol{\omega}' \end{bmatrix}$$

$$\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^E) = \begin{bmatrix} \mathbf{\Phi}(\overline{\boldsymbol{S}}^{\boldsymbol{E}}) & 0 & \dots & 0 \\ 0 & \mathbf{\Phi}(\overline{\boldsymbol{S}}^{\boldsymbol{E}}) & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \mathbf{\Phi}(\overline{\boldsymbol{S}}^{\boldsymbol{E}}) \end{bmatrix} \qquad \mathbf{\Phi}(\overline{\boldsymbol{S}}^{\boldsymbol{E}}) = \begin{bmatrix} \phi_1(\overline{\boldsymbol{S}}^E_{(1,:)}) & \dots & \phi_n(\overline{\boldsymbol{S}}^E_{(1,:)}) \\ \vdots & \dots & \vdots \\ \phi_1(\overline{\boldsymbol{S}}^E_{(mn,:)}) & \dots & \phi_n(\overline{\boldsymbol{S}}^E_{(mn,:)}) \end{bmatrix}$$

$\widehat{\mathbf{\Omega}}$ is of dimension $4n \times 4mn$, $\mathbf{\Omega}$ is $n \times mn$, $\widehat{\mathbf{\Phi}}(\overline{\boldsymbol{S}}^E)$ is $4mn \times 4n$ and $\mathbf{\Phi}(\overline{\boldsymbol{S}}^{\boldsymbol{E}})$ is $mn \times n$. We can now write the system of equations over the grid set $\overline{\boldsymbol{S}}$ in matrix notation:

$$\boldsymbol{A}\,\boldsymbol{Z}(\overline{\boldsymbol{S}}) = \boldsymbol{B}\,E\boldsymbol{Z}(\overline{\boldsymbol{S}}) + \boldsymbol{D}\;\boldsymbol{sx} + \boldsymbol{sz} + \boldsymbol{sv} \tag{46}$$

Or equivalently,

$$\boldsymbol{A}\,\boldsymbol{C} = \boldsymbol{B}\,\widehat{\boldsymbol{\Omega}}\,\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)\,\boldsymbol{C} + \boldsymbol{D}\;\boldsymbol{sx} + \boldsymbol{sz} + \boldsymbol{sv} \tag{47}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{D}$ are matrices that contain the parameters of the model equations. $\boldsymbol{sx}$ is a $4n \times 1$ vector for the grid points of the endogenous state variable $\widehat{s}$ and $\boldsymbol{sz}$ is a $4n \times 1$ vector for the grid points of the exogenous state variable $\widehat{\xi}$. In particular, we have

$$\boldsymbol{sx} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \overline{\boldsymbol{s}} \otimes \boldsymbol{1}_{n_\xi} \end{bmatrix} \qquad \boldsymbol{sz} = \begin{bmatrix} \boldsymbol{1}_{(n_s)} \otimes \overline{\boldsymbol{\xi}} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{48}$$

In each iteration step, we use the current solution for $\boldsymbol{C}$ to check for those grid points where the ZLB is binding. $\boldsymbol{sv}$ is a vector where the elements are equal to $\log \beta$ at the grid points for which the ZLB holds and 0 otherwise. At these grid points, the Taylor rule will be basically "turned off" and the Taylor coefficients become 0. This implies that the parameter matrix $\boldsymbol{A}$ will also be continuously updated.

I use the following iterative approach to find the coefficient vector $\boldsymbol{C}$:

1. Start with a guess $\boldsymbol{C}^0$. Compute $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{sv}$ and $\boldsymbol{A}$ based on $\boldsymbol{C}^0$.

2. Given $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{sv}$ and $\boldsymbol{A}$, solve for $\boldsymbol{C}^{\text{new}}$:

$$\boldsymbol{C}^{\text{new}} = [\boldsymbol{A} - \boldsymbol{B}\,\widehat{\boldsymbol{\Omega}}\,\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)]^{-1}[\boldsymbol{D}\;\boldsymbol{sx} + \boldsymbol{sz} + \boldsymbol{sv}] \tag{49}$$

3. Update $\boldsymbol{C}^1 = \lambda \boldsymbol{C}^0 + (1-\lambda)\boldsymbol{C}^{\text{new}}$ with $\lambda = 0.5$. Then, compute $\overline{\boldsymbol{S}}^E$, $\widehat{\boldsymbol{\Phi}}(\overline{\boldsymbol{S}}^E)$, $\boldsymbol{sv}$ and $\boldsymbol{A}$ based on $\boldsymbol{C}^1$.

4. Redo previous steps until $||vec(\boldsymbol{C}^s - \boldsymbol{C}^{s-1})|| < \delta$. with $\delta = 10^{-8}$ or $s >= 300$.